Letters

Guided Waves on a Flattened Sheath Helix

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Abstract—**The "flattened sheath helix" considered in this letter consists of a pair of parallel unidirectionally conducting screens conducting in different directions and having different dielectric media in the sandwiched and outer regions. Special cases are 1) the normal flattened helix in which the inner medium is a solid dielectric and the outer medium is air, and 2) the inverted flattened helix with the two media interchanged. The guiding properties of such structures are studied.**

Introduction

One of the most important slow-wave structures is the conducting helix of circular cross section. Its properties have been extensively studied and are well documented (see, for example, [1]). A related structure in planar configuration was studied by Arora [2], [3]. This structure, here referred to as a "flattened helix," consists of a pair of parallel arrays of thin closely spaced straight wires conducting in different directions. In this letter, a generalization of the structure described in [2] and [3] is considered in which the media inside and outside the structure have different permittivities. Two special cases are of particular interest and are examined in some detail. First, when the space between the wire screens is a solid dielectric and the outer medium is air (normal helix); second, the complementary case with the air and dielectric regions interchanged (inverted helix).

Characteristic Equations

The geometry of the flattened helix is shown in Fig. 1. The structure is assumed to extend to infinity in the y and z directions of the rectangular coordinate system. In the commonly used "sheath helix" approximation, the top and bottom surfaces are represented as unidirectionally conducting screens. They conduct in directions y' and y'' which make angles α and $-\alpha$, respectively, with the y axis. Correspondingly, they are perfectly insulating in the perpendicular directions z' and z''.

One may assume that the field has no variation in the y direction. Then, as in [2], the general solution may be decomposed into two parts: 1) transverse symmetric (even) and 2) transverse antisymmetric (odd). The characteristic equation for even modes is

$$\frac{(k_1^2/u_1) + (k_2^2/u_2)\coth u_2a}{u_1 + u_2 \tanh u_2a} = \tan^2 \alpha \qquad (1)$$

where $k_1^2 = \omega^2\mu_0\varepsilon_1$, $k_2^2 = \omega^2\mu_0\varepsilon_2$, and u_1 and u_2 are the transverse decay coefficients in the two regions related to the phase constant β in the z direction by

$$\beta^2 = u_1^2 + k_1^2 = u_2^2 + k_2^2. \qquad (2)$$

For decay of the field in the transverse direction, u_1 must be real and positive. The coefficient u_2, on the other hand, may be either real or imaginary; in the latter case, one simply writes $u_2 = jv_2$

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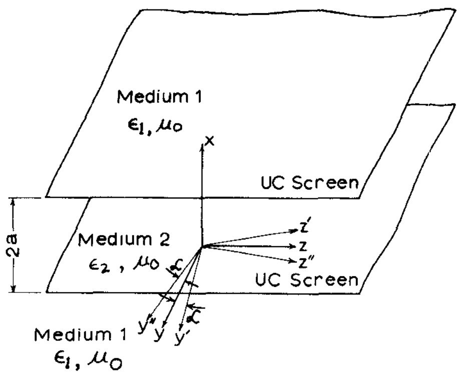

Fig. 1. The flattened sheath helix. z: direction of propagation; y', y'': directions of conduction of the top and bottom unidirectionally conducting screens, respectively; α: helix angle.

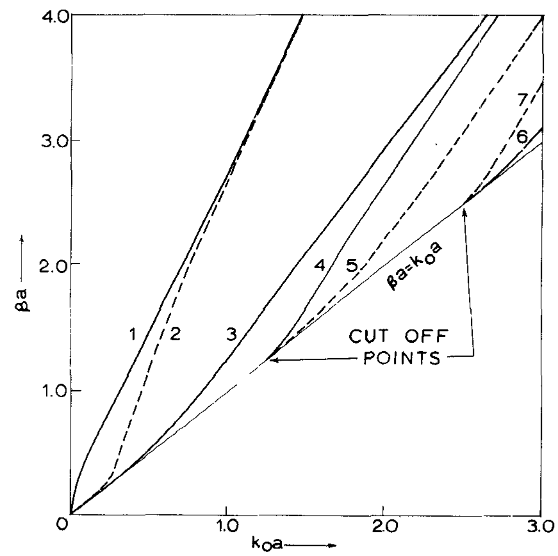

Fig. 2. Dispersion characteristics of a normal helix for $\varepsilon_r = 2.56$ (polystyrene) and $\alpha = 30°$. Continuous lines represent even modes and dotted lines odd modes.

in the preceding equations. The characteristic equation for odd modes is identical to (1) except that the coth and tanh functions are interchanged.

Dispersion Characteristics

The characteristic equations are solved numerically to yield the phase constants for the two sets of modes.

A. Normal Helix

For the case of a normal helix ($\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 = \varepsilon_r\varepsilon_0$, $k_1 = k_0$, $u_1 = u_0$), βa is plotted versus k_0a in Figs. 2 and 3 for $\varepsilon_r = 2.56$ (polystyrene) and $\alpha = 30°$ and $60°$, respectively. Like modes are numbered identically in Figs. 2 and 3. It is found that there are two symmetric modes (modes 1 and 3) and one antisymmetric mode (mode 2) which propagate down to zero frequency. Higher order modes of both symmetric and antisymmetric types

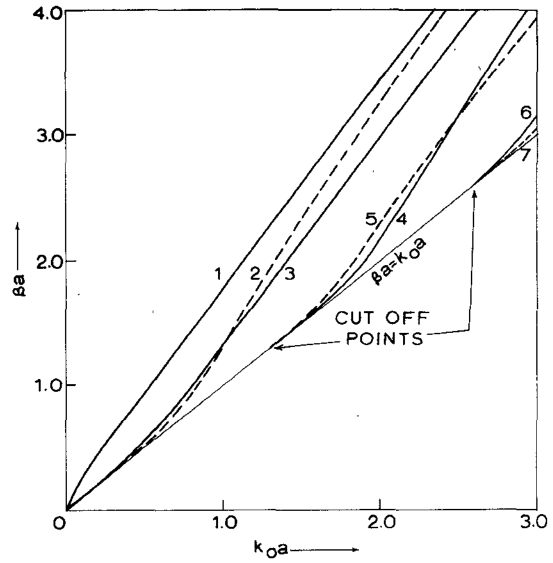

Fig. 3. Dispersion characteristics of a normal helix for $\varepsilon_r = 2.56$ and $\alpha = 60°$. Continuous lines represent even modes and dotted lines odd modes.

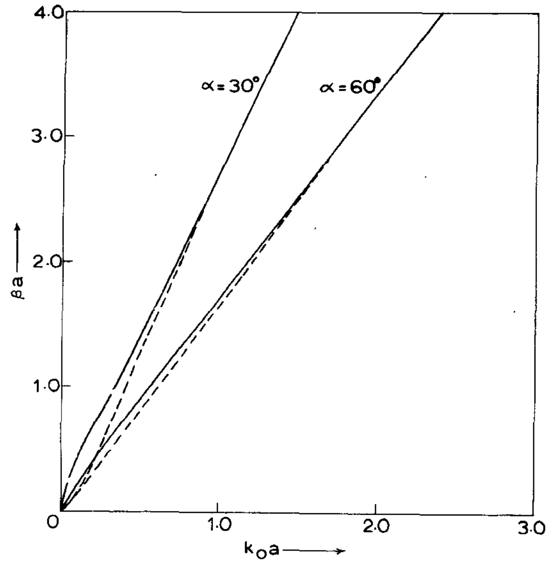

Fig. 4. Dispersion characteristics of an inverted helix for $\varepsilon_r = 2.56$. Continuous lines represent even modes and dotted lines odd modes.

have low-frequency cutoffs given by

$$k_{0c}a = \frac{n\pi}{2(\varepsilon_r - 1)^{1/2}}, \qquad n = 1,2,3,\cdots. \tag{3}$$

At these cutoff frequencies, u_0 vanishes; correspondingly, $\beta = k_{0c}$. It is observed that the curves for modes 1 and 2 tend to merge at higher frequencies. These modes correspond to the two modes which exist when medium 2 also is air [2]. Mode 3, unlike modes 1 and 2, does not have a counterpart in the air case and arises because of the dielectric. Similarly, the presence of higher order modes is also attributed to the dielectric. This conclusion is substantiated by the fact that the modes of the dielectric have cutoff frequencies which also are given by (3) [4].

B. Inverted Helix

In the case of an inverted helix, one sets $\varepsilon_2 = \varepsilon_0$, $\varepsilon_1 = \varepsilon_r\varepsilon_0$, $k_2 = k_0$, $u_2 = u_0$ in (1) and (2). Since, from (2), $u_0^2 = u_1^2 + k_0^2(\varepsilon_r - 1)$ and $u_1 > 0$, $\varepsilon_r > 1$, u_0 is always positive real. Hence there are no modes having trigonometric variation in the central (air) region. A study of the characteristic equations shows that there are only two roots, one symmetric and the other antisymmetric, neither of which has a cutoff frequency. The phase constants are depicted in Fig. 4 as functions of k_0a. These modes are akin to those that exist on a pair of unidirectionally conducting screens in free space [2].

It may be noted that both in the case of normal and inverted helices, there is a single transverse-antisymmetric mode that has no cutoff frequency. By a proper choice of excitation, symmetric modes may be eliminated [3], and the guide dimensions may be chosen in such a way that unimodal propagation occurs.

The electromagnetic field in all the modes is, in general, elliptically polarized both in the longitudinal (xz) and transverse (xy) planes. Further, in all the cases, $E_{y'}$ vanishes in the entire region $x \geq a$ and $E_{y''}$ vanishes in the entire region $x \leq -a$. In the sandwiched region, the electric field vanishes in directions making angles θ_s and θ_a with the y axis for the symmetric and antisymmetric cases, respectively, where these angles are given by

$$\begin{matrix}\theta_s \\ \theta_a\end{matrix} = \tan^{-1}\left(\tan\alpha \begin{matrix}\tanh \\ \coth\end{matrix} u_2a \begin{matrix}\coth \\ \tanh\end{matrix} u_2x\right). \tag{4}$$

In the plane $x = 0$, $\theta_s = \pi/2$, and $\theta_a = 0$. Thus even and odd modes may be distinguished by the manner in which the direction of vanishing electric field component turns while passing from the top to the bottom screen.

Conclusion

A modal analysis has been carried out for the normal and inverted flattened sheath helices. It is shown that by proper geometry and excitation scheme, propagation in a single mode can be assured. The slow wave and polarization properties suggest useful applications for such planar structures. For example, the polarization characteristics may be utilized to fabricate nonreciprocal ferrite components.

References

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Influence of Spatial Dispersion of the Shield Transfer Impedance of a Braided Coaxial Cable

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Abstract—**The effect of the dependence of the braid transfer impedance on the propagation constant is discussed for a coaxial cable located in a circular tunnel.**

In recent papers [1], [2] in this Transactions, we have presented attenuation calculations for a braided coaxial cable located within a circular tunnel bounded by a homogeneous

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